

AdS–Maxwell superalgebra and supergravity

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In this paper we derive the Anti de Sitter counterpart of the super-Maxwell algebra presented recently by Bonanos et. al. Then we gauge this algebra and derive the corresponding supergravity theory, which turns out to be described by the standard $N = 1$ supergravity lagrangian, up to topological terms.

The Maxwell algebra [1], [2] (for recent discussion and references see e.g., [3]) is an extension of Poincaré algebra consisting of the translational and Lorentz generators \mathcal{P}_a and \mathcal{M}_{ab} , respectively appended with six additional generators \mathcal{Z}_{ab} , forming an antisymmetric Lorentz tensor satisfying the relation

$$[\mathcal{P}_a, \mathcal{P}_b] = i\mathcal{Z}_{ab}, \quad (1)$$

with

$$[\mathcal{M}_{ab}, \mathcal{Z}_{cd}] = -i(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}) \quad (2)$$

and $[\mathcal{Z}_{ab}, \mathcal{Z}_{cd}] = 0$, $[\mathcal{Z}_{ab}, \mathcal{P}_c] = 0$. In the paper [5] the authors derive an interesting supersymmetric $N = 1$ extension of this algebra, with two Majorana supercharges.

In the recent paper [4] we extended the the Maxwell algebra (1), (2) to the AdS-Maxwell one (see also [6], [7]) and we derived a dynamical theory resulting from its gauging by using the framework of constrained BF theories [9], [10], [11]. We find that theory obtained by this procedure is just the Einstein-Cartan theory with the additional Holst action term and that the Maxwell field, being the gauge field associated with the generators \mathcal{Z}_{ab} , appears only in the topological term that does not influence the dynamics of the theory. This theory differs therefore from the one discussed in [12]; the reason being that in the latter the Maxwell symmetry was not implemented at the level of the construction of the action.

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In this paper, following the method developed in our earlier work [13], we will construct the $N=1$ supersymmetric extension of the model of [4]. In the first step of the construction we must find the $N = 1$ AdS-Maxwell superalgebra being the supersymmetric counterpart of the AdS-Maxwell algebra [6], [7], [4] with the generators \mathcal{P}_a , \mathcal{M}_{ab} , and \mathcal{Z}_{ab} satisfying the following commutational relations

$$\begin{aligned}
[\mathcal{P}_a, \mathcal{P}_b] &= i(\mathcal{M}_{ab} - \mathcal{Z}_{ab}), \\
[\mathcal{M}_{ab}, \mathcal{M}_{cd}] &= -i(\eta_{ac}\mathcal{M}_{bd} + \eta_{bd}\mathcal{M}_{ac} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad}), \\
[\mathcal{M}_{ab}, \mathcal{Z}_{cd}] &= -i(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}), \\
[\mathcal{Z}_{ab}, \mathcal{Z}_{cd}] &= -i(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}), \\
[\mathcal{M}_{ab}, \mathcal{P}_c] &= -i(\eta_{ac}\mathcal{P}_b - \eta_{bc}\mathcal{P}_a), \\
[\mathcal{Z}_{ab}, \mathcal{P}_c] &= 0.
\end{aligned} \tag{3}$$

The AdS-Maxwell superalgebra, being a supersymmetric extension of (3) contains two supersymmetric generators Q_α and Σ_α , both being Majorana spinors with the following (anti) commutational rules (this algebra in a slightly different form was derived previously in [8])

$$\begin{aligned}
[\mathcal{M}_{ab}, Q_\alpha] &= -\frac{i}{2}(\gamma_{ab} Q)_\alpha, \\
[\mathcal{M}_{ab}, \Sigma_\alpha] &= -\frac{i}{2}(\gamma_{ab} \Sigma)_\alpha, \\
[\mathcal{Z}_{ab}, Q_\alpha] &= -\frac{i}{2}(\gamma_{ab} \Sigma)_\alpha, \\
[\mathcal{Z}_{ab}, \Sigma_\alpha] &= -\frac{i}{2}(\gamma_{ab} Q)_\alpha, \\
[\mathcal{P}_a, Q_\alpha] &= -\frac{i}{2}\gamma_a(Q_\alpha - \Sigma_\alpha), \\
[\mathcal{P}_a, \Sigma_\alpha] &= 0, \\
\{Q_\alpha, Q_\beta\} &= -\frac{i}{2}(\gamma^{ab})_{\alpha\beta}\mathcal{M}_{ab} + i(\gamma^a)_{\alpha\beta}\mathcal{P}_a, \\
\{Q_\alpha, \Sigma_\beta\} &= -\frac{i}{2}(\gamma^{ab})_{\alpha\beta}\mathcal{Z}_{ab}, \\
\{\Sigma_\alpha, \Sigma_\beta\} &= -\frac{i}{2}(\gamma^{ab})_{\alpha\beta}\mathcal{Z}_{ab}.
\end{aligned} \tag{4}$$

By the Wigner-Inönü contraction of the algebra (3) with rescaled generators $\mathcal{P}_a \rightarrow a\mathcal{P}_a$, $\mathcal{Z}_{ab} \rightarrow a^2\mathcal{Z}_{ab}$ and going with a to infinity we obtain the standard Maxwell algebra. As for the supersymmetric extension (4), we rescale $Q \rightarrow a^{1/2}Q$ and $\Sigma \rightarrow a^{3/2}\Sigma$ to obtain the Maxwell superalgebra of [5].

Let us now turn to gauging the AdS-Maxwell superalgebra (3), (4). To this end we write down a gauge field, valued in this superalgebra

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}e_\mu^a\mathcal{P}_a + \frac{1}{2}h_\mu^{ab}\mathcal{Z}_{ab} + \kappa\bar{\psi}_\mu^\alpha Q_\alpha + \tilde{\kappa}\bar{\chi}_\mu^\alpha\Sigma_\alpha \quad (5)$$

In this formula ℓ is a scale of dimension of length necessary for dimensional reason, because the tetrad e_μ^a is dimensionless. Similarly κ and $\tilde{\kappa}$ are scales of dimension $\text{length}^{-1/2}$ included so as to compensate for the dimension of the spinor fields. As we will see below these scales are related. The components of the curvature of connection \mathbb{A}_μ

$$\mathbb{F}_{\mu\nu} = \partial_\mu\mathbb{A}_\nu - \partial_\nu\mathbb{A}_\mu - i[\mathbb{A}_\mu, \mathbb{A}_\nu] \quad (6)$$

can be written as

$$\mathbb{F}_{\mu\nu} = \frac{1}{2}F_{\mu\nu}^{(s)ab}\mathcal{M}_{ab} + F_{\mu\nu}^{(s)a}\mathcal{M}_a + \frac{1}{2}G_{\mu\nu}^{(s)ab}\mathcal{Z}_{ab} + \bar{\mathcal{F}}_{\mu\nu}^\alpha Q_\alpha + \bar{\mathcal{G}}_{\mu\nu}^\alpha\Sigma_\alpha \quad (7)$$

where the supercurvatures are given by

$$\begin{aligned} F_{\mu\nu}^{(s)ab} &= F_{\mu\nu}^{ab} - \kappa^2\bar{\psi}_\mu\gamma^{ab}\psi_\nu, \\ F_{\mu\nu}^{(s)a} &= F_{\mu\nu}^a + \kappa^2\bar{\psi}_\mu\gamma^a\psi_\nu, \\ G_{\mu\nu}^{(s)ab} &= G_{\mu\nu}^{ab} - \tilde{\kappa}\kappa(\bar{\psi}_\mu\gamma^{ab}\chi_\nu + \bar{\chi}_\mu\gamma^{ab}\psi_\nu) - \tilde{\kappa}^2\bar{\chi}_\mu\gamma^{ab}\chi_\nu, \end{aligned} \quad (8)$$

with the bosonic curvatures

$$\begin{aligned} F_{\mu\nu}^{ab} &= R_{\mu\nu}^{ab} + \frac{1}{\ell^2}(e_\mu^a e_\nu^b - e_\nu^a e_\mu^b), \\ \ell F_{\mu\nu}^a &= D_\mu^\omega e_\nu^a - D_\nu^\omega e_\mu^a, \\ G_{\mu\nu}^{ab} &= D_\mu^\omega h_\nu^{ab} - D_\nu^\omega h_\mu^{ab} - \frac{1}{\ell^2}(e_\mu^a e_\nu^b - e_\nu^a e_\mu^b) + (h_\mu^{ac} h_{\nu c}^b - h_\nu^{ac} h_{\mu c}^b). \end{aligned} \quad (9)$$

Notice that the curvature $\ell F_{\mu\nu}^a$ is nothing but the torsion $T_{\mu\nu}^a$.

With the help of covariant derivative defined to be

$$\mathcal{D}_\mu\psi_\nu = \partial_\mu\psi_\nu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\psi_\nu + \frac{1}{2\ell}e_\mu^a\gamma_a\psi_\nu = \mathcal{D}_\mu^\omega\psi_\nu + \frac{1}{2\ell}e_\mu^a\gamma_a\psi_\nu. \quad (10)$$

we can write down the fermionic curvatures in a compact form as follows

$$\begin{aligned} \mathcal{G}_{\mu\nu} &= \tilde{\kappa}\left((\mathcal{D}_\mu^\omega\chi_\nu - \mathcal{D}_\nu^\omega\chi_\mu) + \frac{1}{4}(h_\mu^{ab}\gamma_{ab}\chi_\nu - h_\nu^{ab}\gamma_{ab}\chi_\mu) \right. \\ &\quad \left. + \frac{\kappa}{4\tilde{\kappa}}(h_\mu^{ab}\gamma_{ab}\psi_\nu - h_\nu^{ab}\gamma_{ab}\psi_\mu) - \frac{1}{2\ell}\frac{\kappa}{\tilde{\kappa}}(e_\mu^a\gamma_a\psi_\nu - e_\nu^a\gamma_a\psi_\mu)\right) \\ \mathcal{F}_{\mu\nu} &= \kappa\left(\mathcal{D}_\mu^\omega\psi_\nu - \mathcal{D}_\nu^\omega\psi_\mu + \frac{1}{2\ell}(e_\mu^a\gamma_a\psi_\nu - e_\nu^a\gamma_a\psi_\mu)\right). \end{aligned} \quad (11)$$

Having these building blocks we can proceed to the construction of the action of the AdS-Maxwell supergravity. To this end we generalize the construction presented in [13] including an additional 2-form fermionic field \mathcal{C}^α associated with the supercharge Σ_α . The action reads

$$\begin{aligned}
64\pi\mathcal{L} = & \left(B_{\mu\nu}^{IJ} F_{\rho\sigma}^{(s)}{}_{IJ} - \frac{\beta}{2} B_{\mu\nu}^{IJ} B_{\rho\sigma}{}_{IJ} - \frac{\alpha}{4} \epsilon_{abcd} B_{\mu\nu}^{ab} B_{\rho\sigma}^{cd} \right) \epsilon^{\mu\nu\rho\sigma} \\
& + \left(C_{\mu\nu}^{ab} G_{\rho\sigma}^{(s)}{}_{ab} - \frac{\rho}{2} C_{\mu\nu}^{ab} C_{\rho\sigma}{}_{ab} - \frac{\sigma}{4} \epsilon_{abcd} C_{\mu\nu}^{ab} C_{\rho\sigma}^{cd} \right) \epsilon^{\mu\nu\rho\sigma} \\
& + \left(\beta C_{\mu\nu}^{ab} B_{\rho\sigma}{}_{ab} + \frac{\alpha}{2} \epsilon_{abcd} C_{\mu\nu}^{ab} B_{\rho\sigma}^{cd} \right) \epsilon^{\mu\nu\rho\sigma} \\
& + 4 \left(\bar{\mathcal{B}}_{\mu\nu} \mathcal{F}_{\rho\sigma} - \frac{\beta}{2} \bar{\mathcal{B}}_{\mu\nu} \mathcal{B}_{\rho\sigma} - \frac{\alpha}{2} \bar{\mathcal{B}}_{\mu\nu} \gamma^5 \mathcal{B}_{\rho\sigma} \right) \epsilon^{\mu\nu\rho\sigma} \\
& + 4 \left(\bar{\mathcal{C}}_{\mu\nu} \mathcal{G}_{\rho\sigma} - \frac{\rho}{2} \bar{\mathcal{C}}_{\mu\nu} \mathcal{C}_{\rho\sigma} - \frac{\sigma}{2} \bar{\mathcal{C}}_{\mu\nu} \gamma^5 \mathcal{C}_{\rho\sigma} \right) \epsilon^{\mu\nu\rho\sigma} \\
& + 4 \left(\frac{\beta}{2} \bar{\mathcal{C}}_{\mu\nu} \mathcal{B}_{\rho\sigma} + \frac{\beta}{2} \bar{\mathcal{B}}_{\mu\nu} \mathcal{C}_{\rho\sigma} + \frac{\alpha}{2} \bar{\mathcal{C}}_{\mu\nu} \gamma^5 \mathcal{B}_{\rho\sigma} + \frac{\alpha}{2} \bar{\mathcal{B}}_{\mu\nu} \gamma^5 \mathcal{C}_{\rho\sigma} \right) \epsilon^{\mu\nu\rho\sigma}. \tag{12}
\end{aligned}$$

The bosonic part of this action coincides with the action of AdS-Maxwell gravity derived in [4], while the action (12) with $\mathcal{C} = \mathcal{G} = 0$ is just the $N = 1$ supergravity action in the constrained BF formalism constructed in [13].

Solving the algebraic field equations for the fermionic two form fields we find

$$\begin{aligned}
\mathcal{B} - \mathcal{C} &= \frac{1}{\alpha^2 + \beta^2} (\beta \mathbb{1} - \alpha \gamma^5) \mathcal{F}, \\
\mathcal{C} &= \frac{(\rho - \beta) \mathbb{1} - (\sigma - \alpha) \gamma^5}{(\sigma - \alpha)^2 + (\rho - \beta)^2} (\mathcal{G} + \mathcal{F}), \tag{13}
\end{aligned}$$

which after substituting back to the fermionic part of the action (12) gives

$$\begin{aligned}
16\pi\mathcal{L}^f = & \epsilon^{\mu\nu\rho\sigma} \frac{\alpha}{(\alpha^2 + \beta^2)} \bar{\mathcal{F}}_{\mu\nu} \left(\frac{\beta \mathbb{1} - \alpha \gamma^5}{2\alpha} \right) \mathcal{F}_{\rho\sigma} \\
& + \epsilon^{\mu\nu\rho\sigma} \frac{(\sigma - \alpha)}{(\sigma - \alpha)^2 + (\rho - \beta)^2} (\bar{\mathcal{G}}_{\mu\nu} + \bar{\mathcal{F}}_{\mu\nu}) \left(\frac{(\rho - \beta) \mathbb{1} - (\sigma - \alpha) \gamma^5}{2(\sigma - \alpha)} \right) (\mathcal{G}_{\rho\sigma} + \mathcal{F}_{\rho\sigma}) \tag{14}
\end{aligned}$$

Similarly for the bosonic part of the action we get (see [4] for details)

$$\begin{aligned}
16\pi\mathcal{L}^b = & \epsilon^{\mu\nu\rho\sigma} \left(\frac{1}{\beta} F_{\mu\nu}^{(s)a4} F_{\rho\sigma}^{(s)}{}_{a4} + \frac{1}{4} M^{abcd} F_{ab}^{(s)}{}_{\mu\nu} F_{cd}^{(s)}{}_{\rho\sigma} \right) \\
& + \epsilon^{\mu\nu\rho\sigma} \frac{1}{4} N^{abcd} \left(G_{ab}^{(s)}{}_{\mu\nu} + F_{ab}^{(s)}{}_{\mu\nu} \right) \left(G_{cd}^{(s)}{}_{\rho\sigma} + F_{cd}^{(s)}{}_{\rho\sigma} \right) \tag{15}
\end{aligned}$$

with

$$\begin{aligned}
M^{abcd} &= \frac{\alpha}{(\alpha^2 + \beta^2)} (\gamma \delta^{abcd} - \epsilon^{abcd}), \\
N^{abcd} &= \frac{(\sigma - \alpha)}{(\sigma - \alpha)^2 + (\rho - \beta)^2} \left(\frac{\rho - \beta}{\sigma - \alpha} \delta^{abcd} - \epsilon^{abcd} \right) \tag{16}
\end{aligned}$$

Let us also recall that the parameters of the model α , β , and ℓ are related to the physical coupling constants: Newton's constant G , cosmological constant Λ , and Immirzi parameter γ as follows

$$\frac{\Lambda}{3} = -\frac{1}{\ell^2}, \quad \alpha = \frac{G\Lambda}{3} \frac{1}{(1+\gamma^2)}, \quad \beta = \frac{G\Lambda}{3} \frac{\gamma}{(1+\gamma^2)}, \quad \gamma = \frac{\beta}{\alpha}. \quad (17)$$

Before going further let us pause here for a moment to discuss local supersymmetry transformations that leave invariant the action being the sum of the fermionic (14) and bosonic (15) parts. The gauge transformation of the connection \mathbb{A}_μ are defined to be

$$\delta \mathbb{A}_\mu = \partial_\mu \Theta - i[\mathbb{A}_\mu, \Theta] \quad (18)$$

with

$$\Theta = \frac{1}{2} \lambda^{ab} \mathcal{M}_{ab} + \xi^a \mathcal{P}_a + \frac{1}{2} \tau^{ab} \mathcal{Z}_{ab} + \bar{\epsilon}^\alpha Q_\alpha + \bar{\zeta}^\alpha \Sigma_\alpha. \quad (19)$$

Substituting (19) into (18), decomposing the result and comparing with (5) we find for the local supersymmetry transformations with parameter ϵ

$$\begin{aligned} \delta_\epsilon e_\mu^a &= -\ell \kappa \bar{\epsilon} \gamma^a \psi \\ \delta_\epsilon \omega_\mu^{ab} &= \kappa \bar{\epsilon} \gamma^{ab} \psi \\ \delta_\epsilon h_\mu^{ab} &= \tilde{\kappa} \bar{\epsilon} \gamma^{ab} \chi \\ \delta_\epsilon \bar{\psi}_\mu &= \frac{1}{\kappa} (\mathcal{D}_\mu^\omega \bar{\epsilon} - \frac{1}{2\ell} e_\mu^a \bar{\epsilon} \gamma_a) \\ \delta_\epsilon \bar{\chi}_\mu &= \frac{1}{\tilde{\kappa}} (-\frac{1}{4} h_\mu^{ab} \bar{\epsilon} \gamma_{ab} + \frac{1}{2\ell} e_\mu^a \bar{\epsilon} \gamma_a) \end{aligned} \quad (20)$$

and with the parameter ζ

$$\begin{aligned} \delta_\zeta h_\mu^{ab} &= \bar{\zeta} \gamma^{ab} (\kappa \psi + \tilde{\kappa} \chi) \\ \delta_\zeta \bar{\chi}_\mu &= \frac{1}{\tilde{\kappa}} \mathcal{D}_\mu^{(\omega+h)} \bar{\zeta}, \end{aligned} \quad (21)$$

where the covariant derivative \mathcal{D} is defined in (10).

Similarly we can find the transformation rules for the curvatures, to wit

$$\begin{aligned} \delta_\epsilon F^{(s)a4} &= -\bar{\epsilon} \gamma^a \mathcal{F} \\ \delta_\epsilon F^{(s)ab} &= \bar{\epsilon} \gamma^{ab} \mathcal{F} \\ \delta_\epsilon G^{(s)ab} &= \bar{\epsilon} \gamma^{ab} \mathcal{G} \\ \delta_\epsilon \bar{\mathcal{F}} &= -\frac{1}{4} \bar{\epsilon} \gamma^{ab} F_{ab}^{(s)} - \frac{1}{2\ell} \bar{\epsilon} \gamma_a F^{(s)a4} \\ \delta_\epsilon \bar{\mathcal{G}} &= -\frac{1}{4} \bar{\epsilon} \gamma^{ab} G_{ab}^{(s)} + \frac{1}{2\ell} \bar{\epsilon} \gamma_a F^{(s)a4}, \end{aligned} \quad (22)$$

and

$$\begin{aligned}\delta_\zeta G^{(s)ab} &= \bar{\zeta} \gamma^{ab} (\mathcal{F} + \mathcal{G}) \\ \delta_\zeta \bar{\mathcal{G}} &= -\frac{1}{4} \bar{\zeta} \gamma^{ab} (F_{ab}^{(s)} + G_{ab}^{(s)}).\end{aligned}\tag{23}$$

Using (22) and (23) one can check, using the *1.5* formalism, that the action is indeed invariant under the action of both these local supersymmetries¹. The action is of course also invariant under the bosonic symmetries: the local Lorentz and Maxwell leave it invariant, and details can be found in [4].

Having convinced ourselves that the action is invariant let us try to simplify it. Indeed we expect a lot of cancelations taking place. As we know from [4] the Maxwell gauge field $h_\mu{}^{ab}$ appears in the bosonic action in the topological terms and, as a consequence of this, its superpartner χ should disappear from the action as well. Let us check if this is indeed what is happening.

To see this let us first notice that the curvatures $F^{(s)}$ and \mathcal{F} have exactly the same form as in the $N = 1$ AdS supergravity discussed in [13], so that we must only consider the $F^{(s)} + G^{(s)}$ and $\mathcal{F} + \mathcal{G}$ terms in the lagrangians (14), (15). These terms have the form

$$\begin{aligned}G_{\mu\nu}^{(s)ab} + F_{\mu\nu}^{(s)ab} &= R_{\mu\nu}^{ab}(\omega + h) - \left(\kappa \bar{\psi}_\mu + \tilde{\kappa} \bar{\chi}_\mu \right) \gamma^{ab} \left(\kappa \psi_\nu + \tilde{\kappa} \chi_\nu \right) \\ \mathcal{G}_{\mu\nu} + \mathcal{F}_{\mu\nu} &= \mathcal{D}_\mu^{(\omega+h)} \left(\kappa \psi_\nu + \tilde{\kappa} \chi_\nu \right) - \mathcal{D}_\nu^{(\omega+h)} \left(\kappa \psi_\mu + \tilde{\kappa} \chi_\mu \right).\end{aligned}\tag{24}$$

Using this and

$$\begin{aligned}\bar{\mathcal{F}}_{\mu\nu} \left(\frac{\mathbb{1}\beta - \gamma^5 \alpha}{2\alpha} \right) \mathcal{F}_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} &= \\ &= 4 \frac{\kappa^2}{2} (D_\mu \bar{\psi}_\nu) (\gamma \mathbb{1} - \gamma^5) (D_\rho \psi_\sigma) \epsilon^{\mu\nu\rho\sigma} \\ &= \frac{\kappa^2}{4} \bar{\psi}_\mu (\gamma \mathbb{1} - \gamma^5) \left(\gamma_{ab} F_{\nu\rho}^{ab} + \gamma_a \frac{2}{\ell} T_{\nu\rho}^a \right) \psi_\sigma \epsilon^{\mu\nu\rho\sigma} \\ &\quad + \kappa^2 \bar{\psi}_\mu \left(\frac{1}{\ell^2} \gamma^5 \gamma_{ab} e_\nu^a e_\rho^b + \frac{2}{\ell} \gamma^5 \gamma_a e_\nu^a \mathcal{D}_\rho^\omega \right) \psi_\sigma \epsilon^{\mu\nu\rho\sigma} \\ &\quad + \text{total derivative}\end{aligned}\tag{25}$$

after some straightforward but tedious calculations one can bring the Lagrangian to the following

¹ More precisely, the variation of the action is proportional to super-torsion, which vanishes in the *1.5* formalism.

form

$$\begin{aligned}
16\pi\mathcal{L} = & - \left(\frac{\kappa^2}{G} \bar{\psi}_\mu \gamma^5 \gamma_{ab} e_\nu^a e_\rho^b + \frac{2\kappa^2\ell}{G} \bar{\psi}_\mu \gamma^5 \gamma_a e_\nu^a \mathcal{D}_\rho^\omega \psi_\sigma \right) \epsilon^{\mu\nu\rho\sigma} \\
& - \bar{\psi}_\mu \left(\frac{1}{4\beta} \frac{2\kappa^2}{\ell} \gamma_a T_{\nu\rho}^a + \frac{2\kappa^2\ell}{4G} (\gamma\mathbb{1} - \gamma^5) \gamma_a T_{\nu\rho}^a \right) \psi_\sigma \epsilon^{\mu\nu\rho\sigma} \\
& - \frac{1}{4\beta} \left(\frac{1}{\ell^2} T_{\mu\nu}^a T_{\rho\sigma a} + \kappa^4 \bar{\psi}_\mu \gamma^a \psi_\nu \bar{\psi}_\rho \gamma_a \psi_\sigma \right) \epsilon^{\mu\nu\rho\sigma} \\
& + \frac{1}{16} M_{abcd} \left(F_{\mu\nu}^{ab} F_{\rho\sigma}^{cd} + \kappa^4 \bar{\psi}_\mu \gamma^{ab} \psi_\nu \bar{\psi}_\rho \gamma^{cd} \psi_\sigma \right) \epsilon^{\mu\nu\rho\sigma} \\
& + \frac{1}{16} N_{abcd} (\kappa \bar{\psi}_\mu + \bar{\kappa} \bar{\chi}_\mu) \gamma^{ab} (\kappa \psi_\nu + \bar{\kappa} \chi_\nu) (\kappa \bar{\psi}_\rho + \bar{\kappa} \bar{\chi}_\rho) \gamma^{cd} (\kappa \psi_\sigma + \bar{\kappa} \chi_\sigma) \epsilon^{\mu\nu\rho\sigma} \\
& + \text{total derivative}
\end{aligned} \tag{26}$$

Making use of Fierz identities one can check that the last line in (26) vanishes identically along with other four-fermion terms and there are some simplifications in the second line. Notice that in this way there is no trace of χ in the bulk Lagrangian anymore. Indeed, after some cancelations all the χ -dependent terms can be combined into a total derivative.

The Lagrangian reduces therefore to the final form

$$\begin{aligned}
16\pi\mathcal{L} = & \left(\frac{1}{16} M_{abcd} F_{\mu\nu}^{ab} F_{\rho\sigma}^{cd} - \frac{1}{4\beta\ell^2} T_{\mu\nu}^a T_{\rho\sigma a} \right) \epsilon^{\mu\nu\rho\sigma} \\
& - \left(\frac{\kappa^2}{G} \bar{\psi}_\mu \gamma^5 \gamma_{ab} e_\nu^a e_\rho^b + \frac{2\kappa^2\ell}{G} \bar{\psi}_\mu \gamma^5 \gamma_a e_\nu^a \mathcal{D}_\rho^\omega \psi_\sigma \right) \epsilon^{\mu\nu\rho\sigma} \\
& + \frac{\kappa^2\ell}{2\gamma G} \bar{\psi}_\mu \gamma_a \psi_\nu T_{\rho\sigma}^a \epsilon^{\mu\nu\rho\sigma} + \text{total derivative},
\end{aligned} \tag{27}$$

which, up to the total derivative terms being the supersymmetric generalization of those found in the bosonic case [4], is just the $N = 1$ supergravity Lagrangian, cf. [13] if we set $\kappa^2 = \frac{4\pi G}{\ell}$. Although these topological terms do not change the bulk field equations they may influence the asymptotic charges in an interesting way. We will investigate this in a separate paper.

This concludes our construction, in which we showed that the gauged theory of the AdS-Maxwell supersymmetry is somehow trivial, reducing just to the standard $N = 1$ supergravity. In view of [4] this result is hardly surprising, since in that paper we found that in the bosonic case the gauge field of Maxwell symmetry $h_\mu{}^{ab}$ appears similarly only through a topological term.

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